Communications & Electronics Engineering Dept.

Part 5

Detection & Correction Codes

Communication Networks

(650536)

Prerequisite: Digital Communications (610533)

Instructor: *Dr. Abdel-Rahman Al-Qawasmi*

Text Book

Wireless Communications & Networking.
William Stallings Published by:
Pearson Education, 2002

Coding & Error Control

Three approaches are in common use for copying with data transmission errors:

Error detection codes

Forward Error correction codes

Automatic repeat request.

Error Detection

$$P_1 = (1 - P_b)^F$$ Probability that a frame arrives with no bit errors

$$P_2 = 1 - P_1$$ Probability that a frame arrives with one or more undetected errors

Example: for ISDN (Integrated Services Digital Network):

- BER $=10^{-6}$ and R_b=64 kbps at least 90% of observed 1-minute interval.
- Frame length=1000 bits and number of in a day=64 frame X 60 X 60 X 24=$5.529\text{X}10^6$
- $P_1=(1-10^{-6})^{1000}=0.999$ and $P_2=1-P_1=10^{-3}$ (too large)

We use error detection techniques to decrease P_2

Error Detection

Parity Check: Appending a parity bit to the end of a block of data

Example:

If the transmitter is transmitting 1110001

Sum by modulo 2:

1+1+1+0+0+0+0+1=0 (even parity)

The transmitter will append a 0 and transmit:

11100010

The receiver examines the received character and if the total number is even, assumes no errors. For two errors –Undetected errors.

Cyclic Redundancy Check (CRC): More powerful error detecting codes.

Block of data=k bits	Frame check sequence FCS

Frame to be transmitted T=k+FCK

The pattern P=n – k + 1

In transmitter: the FCS is the reminder (R) of dividing T+(n-k) zero bits by pattern P.

In Receiver : the received frame is to be divided by the pattern P.

-No reminder (No errors)

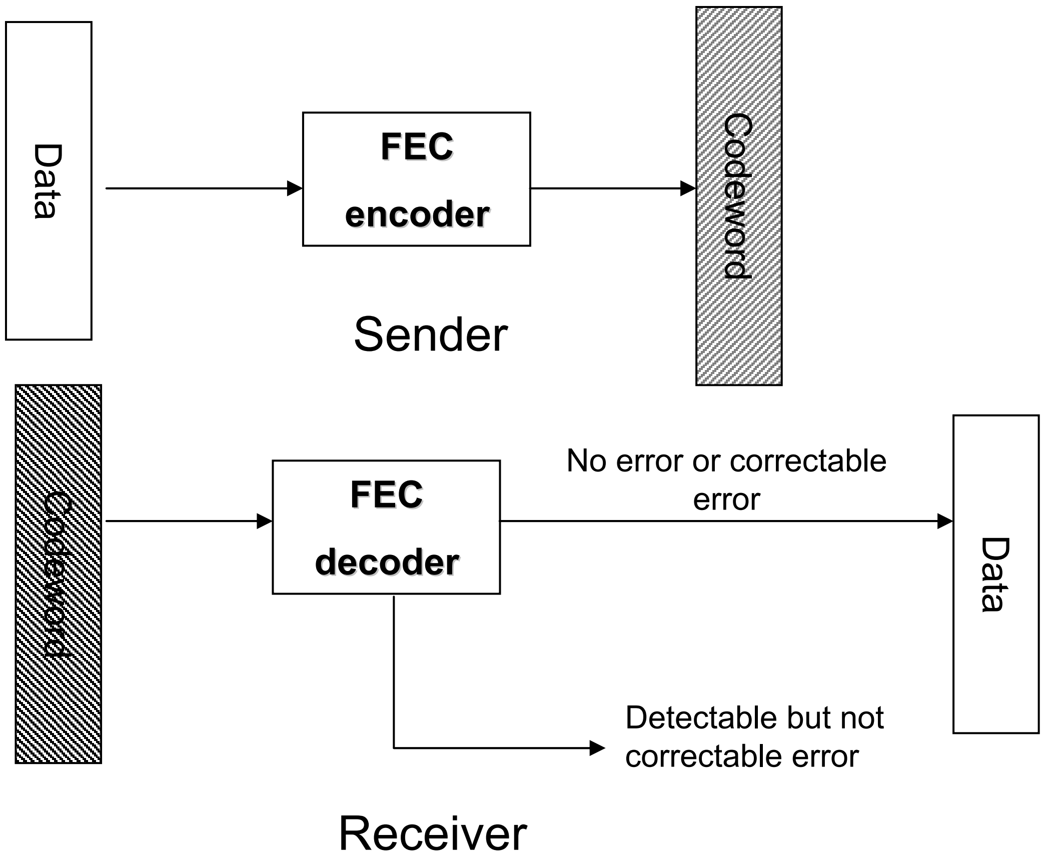
We can view CRC as bits or polynomials

See the examples (Page 208-209)

Block Error Correction Codes

The use of error detection code in wireless applications is not useful for two reasons:

1. The bit error on a wireless link can be high, which would result in a large number of retransmissions.
2. In satellite links the delay is very long for a single frame (inefficient system).



There are 4 possible outcomes of the decoder:

1. **No errors**
2. **Correctable errors**
3. **Not correctable errors**
4. **No errors detected**

Block Code Principles

- **Hamming Distance $d(v_1,v_2)$**: is the number of bits in which v_1 and v_2 disagree.

Example: v_1=100011 & v_2=111011, then $\mathbf{d(v_1,v_2)}$ = 2.

- Instead of transmitting k-bits (data), we map each k-bits into a unique n-bits codeword.

Example:

data block (k=2)	codeword (n=5)
00	00000
01	00111
10	11001
11	11110

- Each valid codeword reproduces the original k data bits and ads to them (n - k) check bits to form the n-bit codeword.
- Number of valid codewords $=2^k$ & Number of all codewords$=2^n$
- Redundancy of the code=(n - k) / k . Code rate=k / n (required additional bandwidth using code)
- The maximum number of guaranteed correctable errors per codeword :

$$t = \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor$$

- The number of errors that can be detected : $t=d_{min} - 1$

Example: if $d_{min}=3$ then number of detected errors= 2 and number of correctable errors=1

Considerations of the design of codeword:

1. Largest possible value of d_{min}
2. Easy code to encode and decode.
3. (n – k) (extra bits) should be small to reduce the bandwidth.
4. (n – k) (extra bits) should be large to reduce error rate.

Hamming Code

Hamming Code parameters: (for m≥3) and as example m=3:

- Block length : $n = 2^m - 1$ = 8 -1 =7
- Number of data bits: $k = 2^m - m - 1$ = 8 – 3 – 1 = 4
- Number of check bits: $n - k = m$ = 3
- Minimum distance $d_{min}=3$ (two detection errors and one correction error)
- The code rate in hamming code= $(2^m - m - 1)\backslash(2^m - 1)$= 4/7 (7/4 bandwidth of uncoded system)
- the syndrome of detection process:

1. If the syndrome contains all 0s, no error has been detected.
2. If the syndrome contains one then , error in check bits.
3. If the syndrome>1 then the numerical value of syndrome indicates the position of error.

- Hamming check bits are inserted at positions that are a power of 2. [2,4,…,$2^{(n-k)}$]
- The remaining bits are data bits.
- To calculate the check bits, each data position which has a value 1 is represented by a binary value equal to its position.
- All of the position values are then XORed together to produce the bits of the Hamming code.
- At receiver all bit position values where there is 1 are XORed.
- The XOR includes data and check bits.